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Cosmological formation of dyon fermion bound states

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Abstract. A calculation of the process of cosmological formation of dyon fermion bound states is reported. It is found that for dyon electron systems the efficiency of bound state formation is about 10% if the dyon change $Q = e/2$ and near total if $Q \geqslant e$. For dyon-proton bound states it is again near total for all $|Q| \geqslant e/2$.

Suppose stable dyons exist in nature. Then the existence of dyon charged-fermion bound states, held together by Coulomb forces, becomes a distinct possibility. The object is very ineresting; its existence would open up entirely new avenues of opportunity concerning monopole search based on observations of the bound state spectrum. This system was studied by us some time ago [1]. A bound state is characterized by the principal quantum number n and the total conserved angular momentum J. The energy levels are [1]

$$E_{n,J} = -13.6\frac{M}{m}\frac{(Q/e)^2}{(n-2|q|+1)^2}\,\mathrm{eV} \qquad J=|q|-\tfrac{1}{2} \tag{1a}$$

$$E_{n,J} = -13.6\frac{M}{m}\frac{(Q/e)^2}{(n-J-|q|-\frac{1}{2}+[J(J+1)+\frac{1}{4}-q^2]^{1/2})^2}\,\mathrm{eV} \qquad J\geqslant|q|+\tfrac{1}{2} \tag{1b}$$

where $M(m)$ is the fermion (electron) mass, $Q(e)$ the dyon (positron) electric charge and $q = eg$, with g the dyon magnetic charge.

Before one can seriously entertain any suggestion concerning a possible experimental search for these objects, it is, of course, essential that one has some assurance that such objects could indeed have been formed, e.g. in the course of cosmological evolution. The process of bound state formation depends critically on the binding energy and thus on the fermion mass and the electric charge and magnetic charge of the dyon. This paper reports a calculation of the cosmological formation of dyon fermion bound states subject to the restriction that the dyon magnetic charge is one Dirac unit, $|q| = \frac{1}{2}$. As for the dyon charge, we shall consider the values $Q = e/2$, $Q = e$ and $Q > e$. The value $Q = e/2$ occurs in the model of Callan [2] and is a generalization of the concept of monopoles with Fermi number one-half discovered by Jackiw and Rebbi [3]. The calculation is done under the following approximation. For dyon electron system we use an expression for the recombination coefficient that consists of the contributions from the process of electron capture into only three types of bound orbits; those with $n = 1$, $J = 0$ (ground state), $n = 2$, $J = 0$ and $n = 2$, $j = 1$. It is clear that the approximation underestimate the efficiency of bound state formation. As regards the bound states of negative dyons with protons, we use the recombination coefficient that is obtained from the dyon–electron case by scaling, i.e. by replacing the electron mass by the proton mass. This approximation does not take into account the effects due to a possible non-minimal electromagnetic interaction due to the proton's anomalous magnetic moment. Our conclusions are as follows. For dyon–electron systems, the fraction of dyons that form into bound states (dyogen) is about 10% when $Q = e/2$ and is close to 100% when $Q \geqslant e$. As for dyon–proton bound states the process of bound state formation proceeds with near 100% efficiency irrespective of the value of Q.

We shall first consider dyon-electron bound state (dyogen) formation for the case where $Q = e/2$. The dyogen binding energy is 3.4 eV. It is clear that dyogen will remain fully ionized during the era of hydrogen recombination. Only well after this era has passed is dyogen formation possible. So the question arises if, during the period of interest, free electrons are available in significant enough amounts to make dyogen formation feasible? From the work of Peebles [4] and Zeldovich *et al* [5] it is known that the residual value of the fractional ionization of hydrogen (ratio of free electron to baryon number densities), at the end of hydrogen recombination era, is of the order 10^{-4} to 10^{-5}. The corresponding free electron density, admittedly very small, must still be very large compared with any dyon density that one can sensibly think of. Thus dyogen formation appears feasible, although we may not expect much efficiency for the process since it takes place under conditions of comparatively low density. The situation is markedly different when $Q \geqslant e$. Now the dyogen binding energy is equal to or greater than 13.6 eV and dyogen formation proceeds contemporaneously with or earlier than the formation of hydrogen. Dyogen formation is expected to be highly efficient under these conditions. A technical distinction between the formation of dyogen as against that of hydrogen may also be noted at this stage. For hydrogen, recombinations direct to the ground state is strongly inhibited, since the recombination photon has a very short mean free path against ionizing another atom. This is not what happens with dyogen, considering the rather low value that the dyon number density must necessarily have. Indeed, the factor that inhibits the recombination of hydrogen direct to the ground state is about 10^7 [4, equation (13)]. The corresponding factor here would be about $10^6\ (n_{\mathrm{D}}/n)$, which cannot be a large number for any thinkable value of the ratio n_{D}/n of dyon to baryon number densities.

We proceed to calculate the formation of dyogen. Let y denote the fractional ionization of dyogen, $y = n_{\mathrm{DF}}/(n_{\mathrm{DF}} + n_{\mathrm{DB}})$, here n_{DF} (n_{DB}) is the number density of free (bound) dyons. Using standard arguments [4, 5, 6] we can derive

$$-\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha\left[n_{\mathrm{e}}y - \frac{1-y}{h^3}(2\pi mkT)^{3/2}\,\mathrm{e}^{-B/kT}\right] \tag{2}$$

where t is the co-moving time, T the blackbody temperature, m the electron mass, $B(=13.6(Q/e)^2$ eV) the dyogen binding energy (ground state); α is the dyogen recombination coefficient and n_{e} the number density of free electrons. The first term on the right-hand side of (2) arises from recombination and the second from photoionization. We note that in the limit of thermal equilibrium ($\mathrm{d}y/\mathrm{d}t = 0$) the expression in parentheses in (2) must vanish and the result is the Saha formula for this process. Equation (2) is valid only when the electrons and the blackbody radiation share a common temperature. When this is no longer true, as indeed it is not at comparatively later times (see below), the equation should modify. Fortunately, it turns out that during this regime the photoionization term is negligibly small as compared with the recombination term. We propose to use the cosmological redshift parameter z as our clock.

We use [5]

$$-\frac{dt}{dz}=\frac{H_0^{-1}}{(1+z)^2(1+\Omega_0 z)^{1/2}}=\frac{3.09\times10^{17}\,h_0^{-1}}{(1+z)^2(1+\Omega_0 z)^{1/2}}\qquad \text{seconds} \tag{3}$$

where $H_0=100\,h_0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the Hubble constant and Ω_0 the present value of the ratio of the mean mass density to the closure density. Further, $T=T_0(1+z)$ with $T_0\simeq3$ K and $n_e=xn$ where x is the fractional ionization of hydrogen and n the (total) baryon number density. Thus

$$n_e=xn_0(1+z)^3\simeq10^{-5}x\Omega_0h_0^2(1+z)^3\ \mathrm{cm^{-3}} \tag{4}$$

where n_0 is the present value of n. Value of $x(z)$ is available from calculations of hydrogen recombination; in particular, [5] gives analytic expressions for the post-hydrogen-formation era, which we shall use. It remains to consider the recombination coefficient α, which is the thermal average of the product of the electron velocity v with the recombination cross section σ. Calculation of cross section for recombination direct to the ground state has been reported elsewhere [7]. Meanwhile, we have computed recombination to the excited states with $n=2$, $J=1$ and $n=2$, $J=0$ (full details will be published separately). We shall, in this paper, work in the approximation in which σ is given by the sum of these three contributions. Further, for the present purpose it seems adequate to compute σ in the limit of very low (incoming) electron velocities. Under these conditions, we find

$$\sigma=7.52\times10^{-6}\frac{(Q/e)^2}{[v/\mathrm{cm\,s^{-1}}]^2}\ \mathrm{cm^2}. \tag{5}$$

And, therefore, the reccombination coefficient is

$$\alpha=\frac{15.4\times10^{-12}}{\sqrt{T_e}}(Q/e)^2\ \mathrm{cm^3\,s^{-1}} \tag{6}$$

where a Maxwellian distribution for electron velocities has been assumed for the calculation of the thermal average of $1/v$ and T_e is the electron temperature in K. We are now in possession of all the needed inputs. Let us first consider the case $Q=e/2$. The calculation of y will be performed in two stages. The first stage covers the period until $z=150$ and the second thereafter. This division is made in conformity with the conclusion of [5] that $T=T_e$ for $z>150$, and $T\neq T_e$ for $z<150$. We choose the values $h_0=1$, $\Omega_0=1$; the expression of $x(z)$ corresponding to this choice is given by [5]

$$x=\frac{1}{33\,100-43z}\qquad z\geqslant150. \tag{7}$$

Using (2)-(7) we obtain

$$\frac{dy}{dz}=\frac{y}{4818-6.26z}+(y-1)\,e^{Y} \tag{8}$$

with

$$Y=50.5-1.5\ln z-\frac{13\,158}{z}. \tag{9}$$

The above equation was solved numerically on a computer. Irrespective of any presumed initial condition, y is found to stay very close to unity until about $z=200$

(reflecting the dominance of the photoionization process), evolve slowly thereafter and reach the value $y = 0.96$ at $z = 150$. At lower redshifts the electron and the blackbody temperatures evolve separately, now $T_e \propto T^2$ and thus

$$T_e = \tfrac{1}{453} T^2 = \tfrac{3}{151}(1+z)^2. \tag{10}$$

The expression for $x(z)$, appropriate to this era, would be [5]

$$x = \frac{1}{38\,348 - 952(1+z)^{1/2}} \qquad z \leqslant 150. \tag{11}$$

Using (10) and (11) in conjunction with (2)-(6) and neglecting the very small contribution of the photoionization term in (2), we obtain

$$\frac{dy}{dz} = \frac{0.09}{(1+z)^{1/2}} \frac{y}{40.3 - (1+z)^{1/2}}. \tag{12}$$

Integrating the above we get the result, valid for $z < 150$,

$$y(z) = y(z=150)\left[\frac{28}{40.3 - \sqrt{1+z}}\right]^{0.18}. \tag{13}$$

Putting in the value $y\ (z = 150) = 0.96$, we thus obtain the asymptotic result $y\ (z = 0) = 0.9$. This means that 10% of the initial assemblage of dyons will convert into dyogen.

Improved estimate of α by taking into account recombinations into final states additional to those considered here will increase α. Clearly, the dyogen fractional concentration will increase as a result. The effect of an increase in the value of dyon charge Q will also be along the same direction (see (5)). Consider now departures from the assumed flat cosmological model, $\Omega_0 = 1$, assumed above. The numerical results of [4] are consistent with the behaviour $x \propto \Omega_0^{-1/2}$, at fixed z. Using this it follows $n_e \propto \Omega_0^{1/2}$ and since $dt/dz \propto \Omega_0^{-1/2}$ for $\Omega_0 z \gg 1$, we see that in the expression for dy/dz the effect of Ω_0 is absent in the recombination term. In this approximation, the net effect is to change (9) by adding the term minus $\ln \sqrt{\Omega_0}$ to the right-hand side. Clearly, the effect is insignificant. In the era $z < 150$ we can no longer necessarily use $\Omega_0 z \gg 1$ and now (3) has to be used without approximation. The effect is to multiply the right-hand side of (11) by the factor $[\Omega_0(1+z)/(1+\Omega_0 z)]^{1/2}$. Evaluating the resulting integral numerically we find $y\ (z=0) \simeq 0.93$ for the case $\Omega_0 = 0.1$. This concludes our discussion of the case $Q = e/2$.

We wish now to consider dyogen formation for the case $Q = e$. For this choice of dyon charge, the dyogen binding energy is 13.6 eV and recombination proceeds contemporaneously with that of hydrogen. From (2)-(6) we get the rate equation

$$\frac{dy}{dz} = 27.51 \Omega_0^{1/2} h_0 \left[xy - \frac{1-y}{\Omega_0 h_0^2} e^{Y_1} \right] \tag{14}$$

with

$$Y_1 = 48.57 - \tfrac{3}{2} \ln z - \frac{52\,632}{z}. \tag{15}$$

The hydrogen fractional ionization $x(z)$, that appears in the above, is to be taken from the calculation of hydrogen recombination [4, 5]; numerical integration of (14) will then yield $y(z)$. However, we wish to avoid taking this route. Our aim here is not so much to precisely map out the evolution of y with z; rather, it is to know what fraction of dyons *eventually* end up recombined. For this purpose, it is adequate to take for $x(z)$ the expression given by the equilibrium Saha formula. Indeed, [5] shows explicitly that in the temperature range of interest, the Saha formula is a good approximation to the actual state of affairs [5, figure 1]. We thus have

$$\frac{x^2}{1-x}=\frac{1}{\Omega_0 h_0^2}\,\mathrm{e}^{Y_1}. \tag{16}$$

On the other hand, when (16) is valid, thermal equilibrium is attained by dyogen recombination as well, as a comparison of the rate equation (14) which the corresponding equation for hydrogen [4, 5] immediately reveals. Thus y is also given by the Saha formula, obtained by setting the right-hand side of (14) equal to zero. It now follows that $x=y$, and from this we conclude that practically all the dyons will end up recombined. Thus, with $\Omega_0 h_0^2=1$ the fractional ionization y has the value 0.01 at $z=1115$. For $\Omega_0 h_0^2=0.1$, this value of y is attained at $z=1050$.

From the foregoing discussion, we draw an important conclusion. Consider higher values of dyon charge $Q>e$. The characteristic era for recombination is earlier than that for hydrogen. Thus $x\simeq 1$. The ratio $\alpha n_e/H$ grows (linearly) with z; here the expansion rate $H=\dot{R}/R$ and R is the scale function of the universe (the *present* value of H is the Hubble constant H_0). The conclusion therefore is that thermal equilibrium would be established. The resulting Saha formula is

$$\frac{y}{1-y}=\frac{1}{\Omega_0 h_0^2}\exp\left[48.57-\tfrac{3}{2}\ln z-\frac{52\,632}{z}\left(\frac{Q}{e}\right)^2\right]. \tag{17}$$

Dyogen recombination is essentially completed when the right-hand side of (17) reaches a value much smaller than unity. For any $Q>e$ we can compute the redshift z at which y attains a very low value, say 0.01.

We shall now consider formation of dyon–proton bound states, for negatively charged dyons. If we ignore the effect due to a possible magnetic interaction arising out of the proton's anomalous magnetic moment, and this cannot alter orders of magnitude, we can obtain the recombination cross section from (5) by scaling, i.e. by multiplying (5) with $(m/M)^2$; here M is the proton mass. An additional factor $\sqrt{M/m}$ enters via thermal averaging and we obtain for the recombination coefficient α_p

$$\alpha_p=1.90\times 10^{-16}\left(\frac{Q}{e}\right)^2\frac{1}{\sqrt{T}}\,\mathrm{cm}^3\,\mathrm{s}^{-1}. \tag{18}$$

The binding energy is also enhanced by the factor (M/m): this is now in the keV range or above. Consequently, the bound state formation takes place at redshifts $z\simeq 10^6$. At these high temperatures, hydrogen remains fully ionized $x=1$. Collecting all these results we have the rate equation

$$\frac{\mathrm{d}y}{\mathrm{d}z_6}=338\Omega_0^{1/2}h_0\left(\frac{Q}{e}\right)^2\left[y-\frac{1-y}{\Omega_0 h_0^2}\,\mathrm{e}^{Y_2}\right] \tag{19}$$

where z_6 is the redshift in units of 10^6 and

$$Y_2=39.13-\tfrac{3}{2}\ln z_6-\frac{96.80}{z_6}\left(\frac{Q}{e}\right)^2. \tag{20}$$

It is a general feature of (19) and (20) that y starts out with the value 1 at suitably high z_6 and approaches fairly rapidly the value zero at around $z_6\simeq 1$. Numerical calculation gives $y=0.01$ at $z_6=2$ for the case $Q=e$ and $y=0.01$ at $z_6=0.5$ for the case $Q=e/2$. These results are for $\Omega_0 h_0^2=1$, other values of this parameter do not change the main conclusion. These results may also be understood on the basis of the Saha formula. The ratio $\alpha_p n_e/H$ is easily computed to be equal to $338 z_6 (Q/e)^2$, and thus thermal equilibrium is a good approximation.

In conclusion, we make several comments. (i) It is natural to expect cosmological formation of dyons with both signs of charge. The negative dyons would all end up forming bound states with protons; positive dyons form bound states with electrons subsequently with efficiency depending on assumed dyon charge. (ii) Relativistic corrections to energy-levels of equation 1*b*) are known [8]. (iii) for dyon-proton bound states (1*a*) and (1*b*) may not be reliable due to the possible presence of an interaction arising out of the proton anomalous magnetic moment [9]. (iv) To what extent do our results depend on the chosen value of the dyon magnetic charge $|q|=\frac{1}{2}$? Recall [1] that for the ground state $J=|q|-\frac{1}{2}$ and $n=2|q|$; thus the binding energy B is independent of q. The value of q enters the calculation of the capture cross section and hence the recombination coefficient. But in all cases except that for dyogen with $Q=e/2$, the bound state formation takes place under conditions of near thermal equilibrium that washes out the information encoded in the recombination coefficient! Thus for these cases our results are insensitive to the choice of q, as also of the various approximations that entered the calculation of the capture cross section. (v) Finally, we comment on the possibility of monopole detection via observations of the spectrum of dyogen. We think of absorption effects in the spectrum of an astronomical body emitting continuous radiation due to the possible presence of dyogen in the intervening medium. Here, the most important absorption lines are those that correspond to transitions from the ground state to final states that can be reached in the dipole approximation. For a dyon magnetic charge of one Dirac unit $|q|=\frac{1}{2}$, and for final states with $J=1$ and principal quantum number n, the absorption lines correspond to wavelengths

$$\lambda=\frac{(n-2+\sqrt{2})^2}{(n-2+\sqrt{2})^2-1}\,\frac{n^2-1}{n^2}\left(\frac{e}{Q}\right)^2\lambda_H \tag{21}$$

where λ_H denotes the wavelength of the corresponding transition in the hydrogen atom (Lyman series). The case $n=2$ is the analogue of the hydrogen Lyman α transition. The absorption coefficients for these lines have been calculated in [1]. Let us consider here the two cases $n=2$ and $n=3$ in some detail. The lines occur at wavelengths $\lambda=1824(e/Q)^2$ Å ($n=2$) and $\lambda=1101(e/Q)^2$ Å ($n=3$) and the corresponding absorption coefficients are $3.98\times10^{-18}(e/Q)^2\ \mathrm{cm}^2$ and $1.01\times10^{-18}(e/Q)^2\ \mathrm{cm}^2$, respectively. Using standard methods [10], we thus obtain for the blue wing of the *redshifted* $1824(e/Q)^2$ Å line the expression of the optical depth

$$\tau(1824(1+z)(e/Q)^2)=3.7\times10^{10}\frac{(1+z)^2}{(1+2q_0 z)^{1/2}}h_0^{-1}\left(\frac{e}{Q}\right)^2\left(\frac{n_{\mathrm{DB}}}{\mathrm{cm}^{-3}}\right) \tag{22}$$

where q_0 is the deceleration parameter, z the redshift of the source and n_{DB} the *present* dyogen number density. To obtain the optical depth for the redshifted $1101(e/Q)^2$ Å line, we have to multiply the right-hand side of (22) by 0.25. To estimate the size of the effect, let us consider that dyons close the universe. Then the dyon number density is roughly on the order $10^{-21}M_{16}^{-1}\ \mathrm{cm}^{-3}$; here M_{16} denotes the dyon mass in units of 10^{16} GeV. Thus, the right-hand side of (22) is on the order $10^{-10}M_{16}^{-1}$. The effect appears too small to be observed if $M_{16}\geqslant 1$, as is the case with certain grand unifying theories (GUTs). There, however, also exist GUTs with an intermediate mass scale; for instance, the Panti–Salam SI(10) model. In the latter, $M_{16} \simeq 10^{-4}$ and it is possible to think of dyons that are even less massive. It is also possible to improve things along another direction; namely, by use of concentrated gas clouds in the intergalactic space. There seems to be some evidence for gas clouds in the absorption *line* spectra (as against an absorbtion trough) of quasi-stellar objects as also in the detailed contour maps of radio sources. Now, in a gas cloud it is not unreasonable to expect a density enhancement of several orders and this, in conjunction with a suitably low value of dyon mass $M_{16} \leqslant 10^{-4}$, would seem to bring the effect close to the realm of observational feasibility. There may very well be other possibilities regarding observations of the spectrum. In particular, observation of the emission spectra is also conceivable.

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